# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2022-23

## MTMACOR11T-MATHEMATICS (CC11)

Time Allotted: 2 Hours — Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates should answer in their own words and adhere to the word limit as practicable.*
*All symbols are of usual significance.*

**Answer Question No. 1 and any *five* questions from the rest**

1. Answer any *five* questions from the following: $2\times 5 = 10$

(a) Form the partial differential equation by eliminating arbitrary functions from the following relation:

$$z=\phi(x+iy)+\psi(x-iy)$$

(b) Solve the following partial differential equation:

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=z$$

(c) Classify the partial differential equation (elliptic, parabolic, or hyperbolic)

$$\frac{\partial^2 u}{\partial x^2}-5\frac{\partial^2 u}{\partial x\partial y}+6\frac{\partial^2 u}{\partial y^2}=0$$

(d) Find the order and degree of the partial differential equations:

(i) $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial x\partial y}+\dfrac{\partial u}{\partial x}-\dfrac{\partial u}{\partial y}=0$

(ii) $\sqrt{1+\dfrac{\partial^2 z}{\partial y^2}}=a\left(\dfrac{\partial z}{\partial x}\right)$

(e) Form the PDE by eliminating $a$, $b$, $c$ from $z=a(x+y)+b(x-y)+abt+c$

(f) State whether the following statement is true or false with reason:
The PDE $x(y+z)p-y(z+x)q+z(x+y)=0$ is quasi-linear.

(g) Prove that $pv=h$ in a central orbit, where the symbols have their usual significance.

(h) A point moves along the arc of a cycloid in such a manner that the tangent at it rotates with constant angular velocity. Show that the acceleration of the moving point is constant in magnitude.

(i) A comet describes a parabola about the Sun. Prove that the sum of the squares of its velocities at the extremities of a focal chord is constant.

2. (a) Find the integral surface given by the equation $x(y^2+z)p-y(x^2+z)q=(x^2-y^2)z$ which contains the straight line $x+y=0$, $z=1$. 5

(b) Find a complete integral of $z=px+qy+p^2+q^2$. 3

3. Solve by the method of separation of variables: 8

$$4\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}-3z=0,\text{ given that } z=3e^{-y}-3e^{-5y}\text{ when } x=0.$$

4. (a) Reduce the partial differential equation $yu_x + u_y = x$ to canonical form and obtain general solution. 4

(b) Obtain the solution of the quasi linear p.d.e. $(y-u)u_x + (u-x)u_y = x-y$ with conditions $u=0$ on $xy=1$ using characteristic equation. 4

5. Solve the one-dimensional wave equation: 8

$$\frac{\partial^2 u}{\partial t^2} - c^2\frac{\partial^2 u}{\partial x^2} = 0,\ t>0$$

subject to the boundary conditions $u(0,t)=0, u(L,t)=0, t>0$ and the initial conditions $u(x,0)=f(x), u_t(x,0)=g(x)$.

6. (a) Find the differential equation of all surfaces of revolution having $z$-axis as the axis of revolution. 4

(b) Find the characteristics of the equation 4

$$y^2\frac{\partial^2 z}{\partial x^2} - x^2\frac{\partial^2 z}{\partial y^2} = 0$$

7. Solve the Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, subject to the condition $u(0,y) = u(l,y) = u(x,0) = 0$ and $u(x,a) = \sin\frac{n\pi}{l}x$ in $0 \le x \le l, 0 \le y \le a$. 8

8. A particle of mass $m$ moves under a central attractive force $m\mu(5r^{-3} + 8c^2r^{-5})$ and it is projected from an apse at a distance $c$ with a velocity $\frac{3\sqrt{\mu}}{c}$. Prove that the orbit is $r = c\cos\frac{2}{3}\theta$. Show further that it will arrive at the origin after a time $\frac{\pi c^2}{8\sqrt{\mu}}$. 8

9. A particle is projected with a velocity $v$ from the Cusp of a smooth cycloid whose axis is vertical and vertex downwards, down the arc. Show that the time of reaching the vertex is 8

$$2\sqrt{\frac{a}{g}}\tan^{-1}\left(\frac{1}{v}\sqrt{4ag}\right)$$

10. The volume of a spherical raindrop falling freely increases at each instant by an amount equal to $\mu$ times its surface area at that instant. If the initial radius of the drop be '$a$', then show that its radius is doubled when it has fallen through a distance $\frac{9a^2g}{32\mu^2}$. 8

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